The Prism-Pair: Simple Dispersion Compensation and Spectral Shaping of Ultrashort Pulses

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Introduction

The generation of ultrashort pulses [1,2] revolutionized the field of molecular spectroscopy [3]. Not only that ultrashort pulses enable to observe molecular motion in time [4-6], but they allow also manipulation of the molecular dynamics [7-9] by controlling the spectral properties of the pulses. For this end, many types of pulse-shaping techniques and configurations are common in spectroscopy-oriented experiments. When complete control of the spectral amplitude and phase is required, a general Fourier-domain pulse-shaper [10] can be used, which provides independent control of both phase and amplitude for each frequency component of the light (e.g. with a spatial light modulator [11,12] or deformable mirrors [13]). However, most applications require only much simpler control of the group delay dispersion (GDD) and higher order dispersion to compensate for the dispersive effect experienced by an ultrashort pulse when passing through optical media and setups. For these applications, the high internal loss and technical complexity of a general pulse-shaper are a burden, and simpler configurations are commonly used, such as the grating-pair [14], chirped mirrors [15] or the Brewster prism-pair [16] - the subject of this mini-review. Prism-pairs can provide tuned compensation with ultra-low loss of up to two orders of dispersion (GDD and sometimes an additional higher order), along with simple amplitude control using a slit or transmission mask in the dispersive arm. Due to the low loss of the prism-pair, it is often the main ‘tool of choice’ for intra-cavity applications [17-19], low light level spectroscopy [20-22], and quantum optics experiments [23-27].

A simple and intuitive formulation is reviewed for the Brewster prism-pair - A most common component in spectroscopy-oriented experiments using ultrashort pulses. This review aims to provide students and beginners in the field of spectroscopy with a unified description of a major experimental component. The total spectral phase experienced by a broadband light field is calculated after passing through a pair of Brewster-cut prisms, demonstrating the flexibility of the prism pair to provide tuned, low-loss control of the dispersion and spectral phase experienced by ultrashort pulses.

Abstract

A simple and intuitive formulation is reviewed for the Brewster prism-pair - A most common component in spectroscopy-oriented experiments using ultrashort pulses. This review aims to provide students and beginners in the field of spectroscopy with a unified description of a major experimental component. The total spectral phase experienced by a broadband light field is calculated after passing through a pair of Brewster-cut prisms, demonstrating the flexibility of the prism pair to provide tuned, low-loss control of the dispersion and spectral phase experienced by ultrashort pulses.

A Single Prism

The analysis starts by reviewing the geometry of a single prism. When a ray passes through a prism at minimum deviation, the angles of refraction through the prism are symmetric [28], as illustrated in figure 1, resulting in:

\[ \theta_1 = \theta_4 = \frac{\alpha}{2} \]

where \( \alpha \) is the apex angle of the prism. If the entrance angle matches with the Brewster angle \( \theta_B \) (for a certain wavelength \( \lambda_0 \)), the refraction angles obey

![Figure 1: Symmetry relations between the refraction angles in a prism. At minimum deviation, the entrance and exit angles are equal and the ray propagates through the prism parallel to its base.](image-url)

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Received: January 27, 2016; Accepted: April 04, 2016; Published: April 07, 2016
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The Optical Path through a Prism-Pair

Our aim now is to calculate the total optical path (and phase) through a prism-pair, which will allow to derive the dispersion experienced by the passing optical pulse. The coordinate system used for the prism-pair has two degrees of freedom (Figure 2): R is the separation between the prisms (segment AB), and H is the penetration of the prism into the beam (segment BC). The red line represents a Brewster ray (at the design wavelength \( \lambda \)) that enters and exits both prisms at Brewster angle \( \theta_B \). The blue line is the deviated ray at \( \lambda \neq \lambda_p \), which deviates from the Brewster ray by an angle \( \delta \theta \) due to the prism dispersion.

Wavefront Calculation

To calculate the phase accumulated by an arbitrary wavelength component \( \lambda \) (blue line) through the prism-pair, one usually calculates the optical path of the deviated beam along a continuous ray, starting from point A, through multiple refractions in the 2nd prism until a final reference plane perpendicular to the beam, located after the 2nd prism where all the colors are parallel \([29,30]\). This method results in somewhat complicated expressions for the optical path, leaving little room for intuition. An elegant alternative (outlined in Figure 3) is to use the concept of wavefronts, the optical path through a non-continuous ray by using the concept of wavefronts, the optical path through the entire system can be reduced to the segment EC.

The same argument applies also to R5, which can be chosen to pass through point C. The conclusion is that the optical path of the deviated ray through the entire prism-pair is simply given by:

\[
P = EC = ED + DC
\]

where \( ED = R \cos \delta \theta \), and \( DC = H \cos \theta_h (\theta_h = \angle DCB) \).

Since \( EC \) is parallel to \( AA' \) and \( R2 \) is parallel to \( R4 \), one can see that \( \theta_h \) is also the angle between \( R2 \) and the deviated ray (blue line) in the 1st prism, as illustrated in figure 5:

\[
\theta_h = \frac{\alpha}{2} - \frac{\pi}{2} - \theta_p - \delta \theta
\]

Substituting Eq. 3 into Eq. 6 obtains:

\[
\theta_h = \pi - 2\theta_p - \delta \theta = \alpha - \delta \theta
\]

indicating that \( DC = H \cos (\alpha - \delta \theta) = H \cos \alpha \cos \delta \theta + H \sin 2\theta_p \sin \delta \theta \).

By substituting the above relations into Eq. 5, the total optical path through the prism-pair is:

\[
P = (R + H \cos \alpha) \cos \delta \theta + H \sin 2\theta_p \sin \delta \theta
\]

and the optical phase experienced by frequency \( \omega \) is

\[
\phi = \omega t
\]

Note that Eq. 8 was a generalization of the method presented in \([31]\) in which only the special case of \( H = 0 \) is presented.

The expressions in equations \([8]\) and \([9]\) for the optical path \( P \) and the optical phase \( \phi \) were expressed as

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and optical phase \( \varphi \) are exact and can be easily used for numerical calculation of the spectral phase \( \varphi(\omega) \) and its frequency derivatives \( GD = d\varphi/d\omega, GDD = d^2\varphi/d\omega^2 \) and higher order dispersion terms. The refraction angle \( \theta_0 \) for each wavelength/frequency can be calculated according to Snell’s law by using the prism’s apex angle and the Sellmeier formula for either \( n(\lambda) \) or \( n(\omega) \).

**Approximation of the Prism Angular Dispersion**

Although Eq. 8 and Eq. 9 already allow complete calculation of the dispersion properties, much intuition can be gained by further developing the expressions with the assumption that the angle \( \delta\theta(\lambda) \) is small. Replacing \( \cos\theta_0 = 1 - \delta\theta^2/2 \) and \( \sin\theta_0 = \delta\theta \) yields:

\[
P = (R + H \cos \alpha) \left( 1 - \delta\theta^2/2 \right) + (H \sin 2\theta_0) \cdot \delta\theta
\]

where \( R = R + H \cos \alpha \) and \( H = H \sin \theta_0 \). The angle \( \theta_0 \) can now be expressed using Snell’s law assuming small angles, such that: \( \cos\theta_0 = 1 \) and \( \sin\theta_0 = \delta\theta \). It is assumed that the beam enters the 1st prism at the minimum deviation angle for a certain wavelength \( \lambda_0 \); that the entrance angle matches the Brewster angle \( \theta_0 \) for \( \lambda_0 \); and that the prism refractive index for \( \lambda_0 \) is \( n(\lambda_0) = n_p \).

The refractive index for the deviated beam inside the prism for the Brewster beam is \( \beta_0 \). The exact spectral phase and any of its derivatives \( (d^n\varphi/d\omega^n) \) can be easily calculated using the chain rule [32]:

\[
dn = \frac{n^2}{dn}  \frac{dn}{d\omega}
\]

For typical optical bandwidths it is safe to assume that \( B \gg 2A\delta n \) and that \( R \gg H \), since \( R \sim 10-30 \text{ cm} \) and \( H \sim 10 \text{ mm} \) are common values. Thus, it is intuitive to think of \( A \) as the distance between the prisms \( (A = R) \), and of \( B \) as the propagation distance inside the prisms. Further approximation can be made since for most practical optical materials \( \sqrt{(dn/d\omega)} \ll \omega^{-1}(dn/d\omega) \). Hence, the total GDD of the prism pair is the sum of two parts: the accumulated GDD from the material dispersion due to propagation through the prisms and an additional negative GDD resulting from the geometric dispersion from the path between the prisms:

\[
\frac{d^2\varphi}{d\omega^2} = \frac{d^2n}{d\omega^2} + \frac{4A}{c} \left( \frac{dn}{d\omega} \right)^2
\]

The geometric dispersion in the second part attributes always a negative GDD for any material (independent of the sign of \( dn/d\omega \)), whereas the first part depends on the material dispersion, which may be either positive (as is usually the case in the visible or NIR range), or negative (as is usually the case for the IR in most materials). Hence, a simple prism-pair offers tunable GDD, both negative and positive, for visible and NIR wavelengths, but for the IR range it will commonly provide only negative GDD. Tuned positive dispersion in the IR range can still be obtained by inserting a 1x1 telescope between the prisms that can flip the sign of the geometric dispersion by imaging the first prism beyond the 2nd prism, effectively generating a “negative distance” \( R \) between the prisms [23].

**Summary**

The performance of the Brewster prism-pair was reviewed - a common major component of ultrafast spectroscopy apparatus. The total spectral phase was calculated as accumulated by broadband light. 

**Citation:** Shaked Y, Yefet S, Pe’er A (2016) The Prism-Pair: Simple Dispersion Compensation and Spectral Shaping of Ultrashort Pulses. Int J Exp Spectroscopic Tech 1:007

**ISSN:** 2631-505X  |  • Page 3 of 4 •
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