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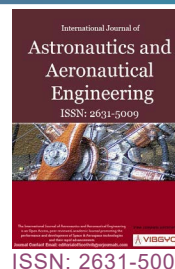
Synthesizes Optimal Conduction Law for Radio Self-Guided Missile Class

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Abstract

The paper considers a method to build optimal conduction law solve the problem of local optimization of the omniscient form based on Letov-Kalman approach. The obtained optimum lead law is similar to the traditional proportional lead law, but the ratio coefficient varies with missile-target distance. The law of conductivity can be realized for the active radio self-guided missile class.

Introduction

In the field of rocket guidance, number of studies suggests different laws of conduction. The law of proportional approach with the fixed coefficients (traditional proportional conduction law) has been widely applied since its high realization ability. However, the paper [1,2] has shown that this lead law has many limitations in the case of self-guided missiles with maneuverable targets. Therefore, the law of conduction has been continuously studied to find a new law of conduction that can improve the quality of missile guidance in the case of mobile targets [3].

Optimized Control Algorithm According to Local Standards

Consider a linear control system with a defined structure as follows:

$$\dot{\mathbf{x}}_y(t) = \mathbf{F}_y(t)\boldsymbol{x}_y(t) + \mathbf{B}_y(t)\boldsymbol{u}(t) + \boldsymbol{\xi}_y(t) \tag{1}$$

Where: $\mathbf{x}_y$ - the output state vector of the system

$\boldsymbol{\xi}_y$ - the systemic noise vector of the Gaussian white noise with the mathematical expectation of 0;

$\mathbf{u}(t)$- control signal vector

$\mathbf{F}y$, $\mathbf{B}y$ -Control efficiency matrix.

The index function of the local quality:

$$I = \left[x_T(t) - x_y(t) \right]^T Q \left[x_T(t) - x_y(t) \right] - \int_0^t U^T(t) K u(t) dt \tag{2}$$

Which: X_T - the request state vector

Q- matrix of penalty coefficients according to accuracy at time t

K- matrix of the penalty coefficients according to the magnitude of the control signal

Solving the problem of finding optimal control signals for system (1) and slab (2) by Bellman dynamic planning method.

$$u = K^{-1}B_y^T Q \left[x_T - x_y \right] \tag{3}$$

Equations of State Describe Self-Conduction Kinetics in Space

The spatial self-conducting geometry is described by the vector equation

$$\ddot{\vec{\lambda}} = \left[-\frac{2\dot{R}}{R} \right]\dot{\vec{\lambda}} + \left[-\frac{\ddot{R}}{R} \right]\vec{\lambda} - \frac{1}{R}\vec{a} + \frac{1}{R}\overrightarrow{a_{mt}} \tag{4}$$

Which: $\overrightarrow{a_{mt}}$, $\vec{a}$ is the total acceleration vector of the target and missile $\vec{\lambda}$ is the unit vector of missile-target distance vector with $\vec{R} = R \cdot \vec{\lambda}$

To describe the coordinates of the vectors $\vec{\lambda}$, $\dot{\vec{\lambda}}$, $\ddot{\vec{\lambda}}$, $\vec{a}$, $\overrightarrow{a_{mt}}$ in the fixed ground coordinate system (inertial coordinate system) we use vectors. The corresponding state silk is as follows:

$$\left\{ \begin{array}{l} \lambda_q = \left[\lambda_{xq}\lambda_{yq}\lambda_{zq} \right]^T ; \ \dot{\lambda}_q = \left[\dot{\lambda}_{xq}\dot{\lambda}_{yq}\dot{\lambda}_{zq} \right]^T ; \ddot{\lambda}_q = \left[\ddot{\lambda}_{xq}\ddot{\lambda}_{yq}\ddot{\lambda}_{zq} \right]^T \\ a_q = \left[a_{xq}a_{yq}a_{zq} \right]^T ; \ a_q^{mt} = \left[a_{xq}^{mt}a_{yq}^{mt}a_{zq}^{mt} \right]^T ; \end{array} \right\} \tag{5}$$

The state vector describes the instantaneous state of the line of sight:

$$X_y = \left[\lambda_q^T \quad \lambda_q'^{T} \right]^T \tag{6}$$

Put:

$$F_y = \begin{bmatrix} O_3 & I_3 \\ -\dfrac{\ddot{R}}{R}I_3 & -\dfrac{2\dot{R}}{R}I_3 \end{bmatrix}; \ \mathbf{B}_y = -\frac{1}{R}\begin{bmatrix} O_3 \\ I_3 \end{bmatrix}; \ u = aq; \ \mathbf{B}_{mt} = \frac{1}{R}\begin{bmatrix} O_3 \\ I_3 \end{bmatrix}; \ \xi_{mt} = a_q^{mt} \tag{7}$$

In which: O_3- is a square matrix of order 3, including only zeros; I_3 is a 3 × 3 unit matrix.

From equations (5) to (7) we have self-conducting geometry described in terms of equations of state

$$\dot{x}_y = \mathbf{F}_y x_y + \mathbf{B}_y u + \mathbf{B}_{mt}\xi_{mt} \tag{8}$$

Select the Optimal Slab and Determine the Optimal Lead Law

Select the optimal slab as follows:

$$I = q_h \cdot \omega_\lambda^2(t) + \int_0^1 K_u \cdot a^2(t)dt \tag{9}$$

In which: $q_h > 0$ is the penalty coefficient according to the instantaneous slip; $k_u > 0$ is the penalty factor for the total acceleration (or overload) required up to time t.

Vector depicts the desired state of sight

$$x^T = \left[O_q^T \quad T \right]^T \tag{10}$$

Put: $Q_h = q_h \begin{bmatrix} O_3 & O_3 \\ O_3 & I_3 \end{bmatrix}$; $\mathbf{K}_u = k_u.I_3$ \hfill (11)

Then the (9) equivalent of the slab

$$I = \left[x_T(t) - x_y(t) \right]^T Q_h \left[x_T(t) - x_y(t) \right] - \int_0^1 u^T(t)K_u u(t)dt \tag{12}$$

When synthesizing the conduction law in the absence of information about the target acceleration,

the target acceleration can be considered as an unknown random effect. The state equation without the impact of the target acceleration has the form

$$\dot{x}_y \;=\; F_y x_y \;+\; B_y u \tag{13}$$

Applying optimal control algorithm for system (13) and local criteria blade (12) we have

$$u \;=\; a_q \;=\; \left(\frac{1}{R}\cdot\frac{q_h}{k_u}\right)\dot{\lambda}_q \tag{14}$$

From (5) and (14) we have optimal lead law corresponding to the criterion (9) or (12)

$$\vec{a} \;=\; \left(\frac{1}{R}\cdot\frac{q_h}{k_u}\right)\dot{\lambda} \tag{15}$$

The symbol $\overrightarrow{\omega_\lambda}$ is the vector of the angle of rotation of the line of sight, we have $\overrightarrow{\omega_\lambda} \perp \vec{\lambda}$ and

$$\dot{\vec{\lambda}} \;=\; \left(\vec{\omega}_\lambda x \vec{\lambda}\right) \;=\; \left(\omega_\lambda . \overrightarrow{\omega_\lambda^0} x \vec{\lambda}\right) \;=\; \omega_\lambda . \overrightarrow{\lambda^0} \tag{16}$$

With $\overrightarrow{\lambda^0}$, $\overrightarrow{\omega_\lambda^0}$ are unit vectors of $\vec{\lambda}$ and $\vec{\omega}_\lambda$ respectively

From (16) we have $\left(\vec{\lambda}, \dot{\vec{\lambda}}, \vec{\omega}_\lambda\right)$ perpendicular to each other and set up the perpendicular coordinate system in space, so we can describe the direction of the required acceleration vector of the name. The rocket corresponds to the law of missile guidance (15) as shown in the Figure 1.

According to [4] the line of sight kinetics is described by the equation

$$R.\dot{\omega}_\lambda \;+\; 2\dot{R}.\omega_\lambda \;=\; \left(a_{mt}^\perp \,-\, a^\perp\right) \tag{17}$$
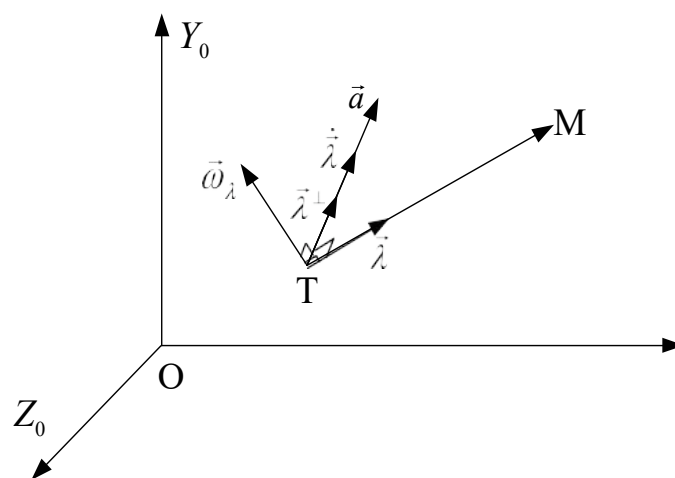
In which: $a_{mt}^\perp$, $a^\perp$ are the projections of the target acceleration vector, the missile is perpendicular to the line of sight and lies in the self-guided plane.

From the picture and according to (16) we have

$$a^\perp \;=\; \vec{a}.\overrightarrow{\lambda^0} \;=\; \left(\frac{1}{R}\cdot\frac{q_h}{k_u}\right)\omega_\lambda \tag{18}$$

Replace (18) into (17) and transform we have

$$\frac{R}{\left(\dfrac{1}{R}\cdot\dfrac{q_h}{k_u}\right)-2V_c}\cdot\omega_\lambda \;+\; \omega_\lambda \;=\; \frac{a_{mt}^\perp}{\left(\dfrac{1}{R}\cdot\dfrac{q_h}{k_u}\right)-2V_c} \tag{19}$$

Figure 1: The required acceleration vector direction of the rocket in space.

In which: $V = -\dot{R}$ is the speed at which the missile approaches the target

According to [5], the instantaneous slip h of the self-conductive process is determined by

$$h = \frac{R^2}{\Delta V} \cdot \omega_\lambda \qquad (20)$$

Where ΔV is the magnitude of the vector $\overrightarrow{\Delta V} = \left(\vec{V} - \overrightarrow{V_{mt}} \right)$

The instantaneous slip (h) of the self-conduction process is proportional to the rotation speed of the line of sight so allowing the self-guided control ring to ensure that the instantaneous slip h → 0 will correspond to the control of the rocket maneuver so that ω_λ → 0. Equation (19) shows that when the missile is self-conducting according to the law (15), the self-conducting control ring corresponds to the first order of inertia stage with the time constant:

$$\tau_{\omega k} = \frac{R}{\dfrac{1}{R} \cdot \dfrac{q_h}{k_u} - 2V_c} \qquad (21)$$

When a missile attacks a mobile target, the total active self-driving time is usually very short. Therefore, to ensure the required slip at the meeting point, we set a requirement to limit the transient time of the self-conducting control ring right from the start of self-conduction:

$$\tau_{\omega k} \leq T_{cp} \qquad (22)$$

Tcp- allowable limit of self-conductive control loop time constant.

From (21) and (22) we have the boundary condition of (22) being equivalent

$$\frac{q_h}{k_u} = 2RV_c + \frac{R^2}{T_{cp}} \qquad (23)$$

For (qh, ku) choose according to (23) then the law of leading (15) becomes

$$\vec{a} = \left(2V_c + \frac{R^2}{T_{cp}} \right) \dot{\vec{\lambda}} \qquad (24)$$

The law of conductivity (24) ensures that the transient time constant of the self-conductive control loop is equal to the limit of the permissible value. Unlike the traditional proportional conduction law, this law of conduction has a proportional coefficient that decreases with distance and depends on the time constant limit [6].

The Simulation Results Evaluate the Optimal Lead Law

Simulations are performed with traditional proportional conduction law (coefficient K = 3) and optimal conduction law (24) under the same conditions: $R_0^p = 6$ [km]; target overload nmt = 9, flight time tq0 = 0.3 [s] Figure 2, Figure 3 and Figure 4.

Comment

• When the target is strongly mobile, the slip at the meeting point of the orbit corresponding to the law of conduction (24) is significantly smaller than the slip at the meeting point of the conduction trajectory corresponding to the traditional proportional conduction law.

• With traditional proportional conduction law, at the beginning of self-conductivity, the required missile overload is of small value so that the initial slip is not quickly eliminated from the start thanks to the maximum effective overload of the name fire.

• With the law of conductivity (24), because slip is always required to reduce quickly with limited transition time, the day from the start of self-conductivity, according to the requirements of the law of conductivity, the missile using the overload is maximized to quickly reduce the initial slip. Therefore the initial slip is reduced faster and the slip at meeting point is guaranteed to be small enough as required.
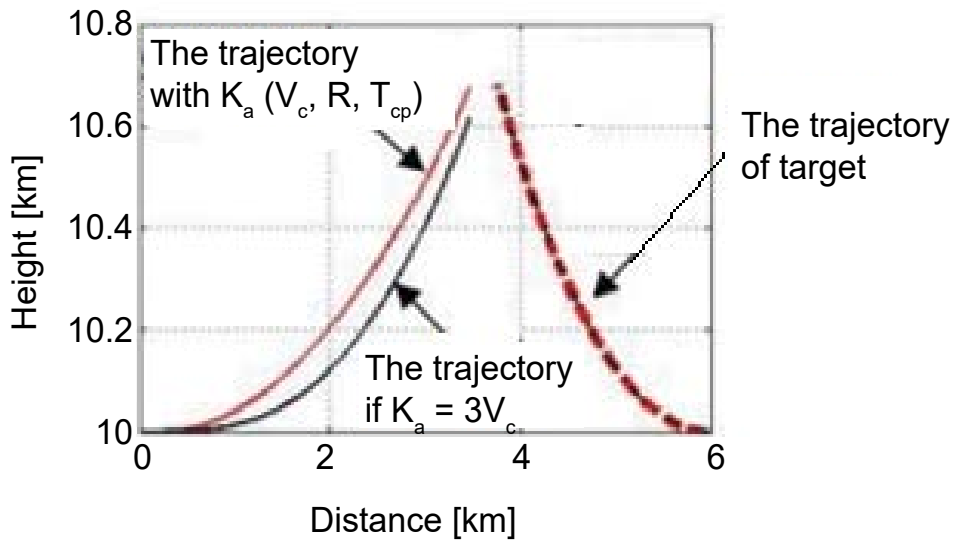
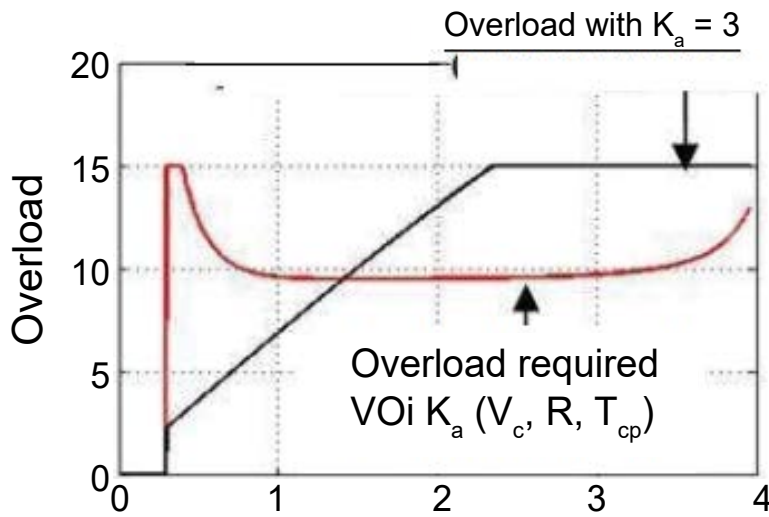**Figure 2:** The lead trajectories of the two laws lead to survey.

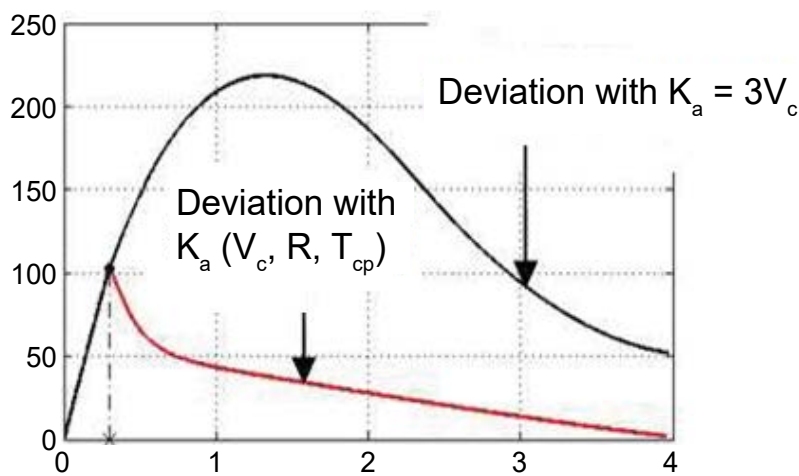**Figure 3:** Overload required by two laws lead.

Figure 4: Slippage of two lead trajectories.
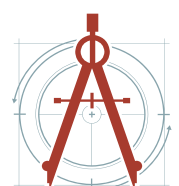
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Conclusion

The survey simulation results show that the obtained optimal conduction law meets the quality requirements of the self-conductive control loop and is more effective than the traditional ratio approach law when the target is maneuverable. This law of conductivity can be applied to the active radio self-guided missile class, which helps to improve the conductivity of these missiles in aerial combat with highly maneuverable flying targets.

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